Effect of long-range interactions on the scaling of the noisy Kuramoto-Sivashinsky equation

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The effects of long-range interactions on the scaling properties of the noisy Kuramoto-Sivashinsky (KS) equation are studied by the dynamic renormalization-group technique. It is found that the presence of long-range nonlinearity in the KS equation can produce new stable fixed points with varying critical exponents that depend on both the long-range interaction parameter ρ and the substrate dimension d.

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I. INTRODUCTION

The formation and kinetic roughening of nonequilibrium interface not only is of practical importance in crystal growth, but also is related to the nonequilibrium statistical physics. Therefore, recently there has been much interest in this field [1–4]. A common feature of many interfaces observed experimentally and in discrete growth models is that their roughening follows simple scaling laws [5]. The morphology and dynamics of a rough interface can be characterized by the surface width, $W(L,t)$, that scales as

$$W(L,t)=\frac{1}{\sqrt{L}}\left\langle \sum_{\mathbf{r}} [h(\mathbf{r},t)-\overline{h}_L(t)]^2\right\rangle^{1/2}=L^{\chi}f\left(\frac{t}{L^z}\right),$$

where χ is the roughness exponent for the interface height $h(\mathbf{r},t)$ and the dynamic exponent z describes the scaling of the relaxation time with the system size L; $\overline{h}_L=(1/L)\Sigma_{\mathbf{r}}h(\mathbf{r},t)$ is the mean height of the interface at time t and the angular brackets denote a noise average. The scaling function f has asymptotic properties such that $W(L,t)\sim t^{\chi/z}$ for $t\ll L^z$ and $W(L,t)\sim t^{\chi}$ for $t\gg L^z$. The scaling exponents χ and z determine the asymptotic behavior of growing interfaces on a large distance and long time scale, and the universality class they belong to. One of the widely used methods of getting these scaling exponents is applying a numerical or analytical approach to the associated stochastic evolution equations that describe the interface growth processes. A seminal example of this kind of stochastic dynamic equations [6] is the well-known Kardar-Parisi-Zhang (KPZ) equation, which has been studied intensively by analytical and numerical methods and a number of theoretical results have been obtained [1–4,7].

The analytical tool now widely used in the analysis of the scaling behaviors of these nonlinear Langevin-type equations is the dynamic renormalization-group (DRG) technique, which was first proposed by Forster, Nelson, and Stephen in the study of the stochastic version of the Burgers equation [8] and was applied to the analysis of the KPZ equation by Kardar and co-workers [6,7]. Currently, the KPZ equation has become a well-known model of dynamic critical phenomena for a large variety of growth problems. On the other hand, only poor agreement between the theoretical results and the experiments [2–4] is observed. As a result, various modifications of the KPZ equation have been proposed [2–4,7].

The KPZ equation has a nonlinear term of short-range(or local) nature describing the lateral growth [6]. In many growth problems, however, the long-range interactions, e.g., the long-ranged hydrodynamic interactions, are necessary [9,10]. In order to incorporate these long-range interactions into the kinetic roughening of surface growth, Mukherji and Bhattacharjee [11] proposed a phenomenological equation with a nonlinear term of long-range nature capable of correlating each site of the growing surface with all other sites. By a DRG analysis, they show that the nonlocal nonlinearity introduced is sufficient to yield new fixed points with continuously varying exponents depending on the long-range feature, and several distinct phase transitions not found in the original KPZ theory. Working along this line and using the DRG method, Jung *et al.* [12] studied the effect of long-range interactions on the conserved KPZ equation and Chattopadhyay [13] investigated the scaling of the nonlocal KPZ equation with spatially correlated noise. They all obtained interesting results.

In addition to the KPZ equation, another nonlinear Langevin equation is the so-called Kuramoto-Sivashinsky (KS) equation [14,15], which has been also actively discussed in the evolution of interface. Cuerno and co-workers [16,17] argued that the noisy KS equation can well describe the evolution and the scaling properties of interface ion-sputtered at normal incidence. Cuerno and Lauritsen [18], and Drotar, Zhao, Lu, and Wang [19] analyzed the noisy KS equation by a DRG analysis and a numerical calculation, respectively.

In the present work, we study the effects of long-range interactions on the scaling behaviors of the noisy KS equation by applying a DRG approach to the noisy KS equation with nonlocal nonlinearity. Our calculations and analyses show that the long-range interactions in the noisy KS equation can also produce new fixed points with the varying dynamic scaling exponents depending on both the long-range interaction strength ρ and the substrate dimension d.

The outline of this paper is as follows. In the next section, we introduce the generalized noisy KS equation with the nonlocal nonlinearity and make a simple scaling analysis. Then we derive the RG flow in Sec. III. Section IV contains our calculations and discussion to the RG flow. The conclusions are given in Sec. V.

II. THE NOISY KS EQUATION WITH LONG-RANGE NONLINEARITY

The noisy KS equation is given by

$$\frac{\partial h(\mathbf{r},t)}{\partial t}=\nu\nabla^2 h-\kappa\nabla^4 h+\frac{\lambda}{2}(\nabla h)^2+\eta(\mathbf{r},t), \tag{1}$$

where $\eta(\mathbf{r},t)$ is the Gaussian white noise with zero mean and short-range correlations described by

$$\langle\eta(\mathbf{r},t)\eta(\mathbf{r}',t')\rangle=2D\delta^d(\mathbf{r}-\mathbf{r}')\delta(t-t'). \tag{2}$$

The parameter ν is generally negative for the noisy KS equation in contrast to the KPZ equation [6,7] in which the Laplacian term has a positive coefficient and corresponds to a surface tension, whereas κ is a positive surface diffusion coefficient [18]. The nonlinear term is the KPZ-type. The strength of the nonlinearity is given by λ as in the KPZ equation [6,7]. The fact $\nu<0$ means that the system is linearly unstable, a fact, which in ion-sputtered systems is related to faster erosion velocity at the bottom of the troughs than at the peaks of the crests, which in turn is related to the formation of the periodic ripple structure [18]. In the KS equation, the combination of $\nabla^2 h$ and $(\nabla h)^2$ terms models the effect of particles being knocked out of the interface by the bombarding ions. These terms can be derived from a simple model of ion bombardment in which the particles are assumed to penetrate a fixed distance into the interface and then spread their energy out with an asymmetric, three-dimensional Gaussian distribution [16]. The $-\nabla^4 h$ term models the effect of surface diffusion [20]. The noise term is present due to the randomness in the arrival of bombarding ions at the interface [16]. The $2D$ in Eq. (2) refers to the variance of the noise term and is proportional to the rate of bombardment [16]. The KS equation is believed to encompass many of the features that are actually realized in ion-sputtering experiments, such as ripple formation and KPZ-type scaling [21]. In principle, the noisy KS equation can appear in any physical system modeled by the deterministic KS equation in which the relevance of time-dependent noise as, e.g., fluctuation in a flux or thermal fluctuations, can be argued for [18].

The nonlocal KPZ equation proposed by Mukherji and Bhattacharjee [11] is the following:

$$\frac{\partial h(\mathbf{r},t)}{\partial t}=\nu\nabla^2 h(\mathbf{r},t)+\eta(\mathbf{r},t)+\frac{1}{2}\int d\mathbf{r}'\,\vartheta(\mathbf{r}')\times\nabla h(\mathbf{r}+\mathbf{r}',t)\cdot\nabla h(\mathbf{r}-\mathbf{r}',t), \tag{3}$$

where $\vartheta(\mathbf{r})$ is taken to have a short-range (SR) part $\lambda_0\delta(\mathbf{r})$ and a long-range (LR) part $\sim r^{\rho-d}$ or more precisely, in Fourier space, $\vartheta(k)=\lambda_0+\lambda_\rho k^{-\rho}$. Equation (3) can smoothly [6] go over to the KPZ equation for $\lambda_\rho=0$, and so all standard KPZ results can be expected for $\lambda_\rho=0$.

Here we extend the phenomenological equation of Mukherji and Bhattacharjee to the noisy KS equation

$$\frac{\partial h(\mathbf{r},t)}{\partial t}=\nu\nabla^2 h(\mathbf{r},t)-\kappa\nabla^4 h(\mathbf{r},t)+\eta(\mathbf{r},t)+\frac{1}{2}\int d\mathbf{r}'\,\vartheta(\mathbf{r}')\nabla h(\mathbf{r}+\mathbf{r}',t)\cdot\nabla h(\mathbf{r}-\mathbf{r}',t), \tag{4}$$

as the starting point of our analysis. It can be seen that both Eqs. (3) and (4) have the long-range interaction in common, and Eq. (4) can shrink to the original noisy KS equation (1) for $\lambda_\rho=0$. If viewed only from mathematical expression, the nonlocal noisy KS equation (4) contains the nonlocal KPZ equation (3) as a special case of $\kappa=0$. In addition, the previously studied Sun-Guo-Grant [22] and molecular-beam-epitaxy [23] equations are also contained in Eq. (4). Thus our analyses and discussion have general interests.

Simple scaling from $\mathbf{r}\to b\mathbf{r}$, $h\to b^\chi h$, and $t\to b^z t$ shows that both the short- C_{SR} and long-ranged C_{LR} contributions in the interaction kernel are relevant for $d<d_c$ (where by C_{SR} interaction we mean the standard KPZ type nonlinearity, and the C_{LR} interaction implies a non-KPZ-type $\mathbf{r}$-dependent part). Under this scale transformation the parameters in Eq. (4) change by $\nu\to b^{z-2}\nu$, $\kappa\to b^{z-4}\kappa$, $D\to b^{z-d-2\chi}D$, $\lambda_0\to b^{\chi+z-2}\lambda_0$, and $\lambda_\rho\to b^{\chi+z-2-\rho}\lambda_\rho$. These scale transformations will be used in the following DRG calculation. In the next section, we will carry out a DRG analysis to Eq. (4) to determine the effect of long-range interaction on the scaling behaviors of the noisy KS equation.

The DRG method is based on an expansion in powers of the nonlinear coupling and a subsequent term by term average over the noise, implementing the statistical average. In the divergent regimes the expansion is regulated by momentum shell integration in the short wavelength limit [6–8].

III. DRG FLOW FOR THE NONLOCAL NOISY KS EQUATION

The DRG procedure can be succinctly described through the Fourier modes momentum $\mathbf{k}$ and frequency ω, in terms of which Eq. (4) becomes

$$h(\mathbf{k},\omega)=G_0(\mathbf{k},\omega)\eta(\mathbf{k},\omega)-\frac{1}{2}G_0(\mathbf{k},\omega)\int_{-\infty}^{+\infty}\frac{d\Omega}{2\pi}\int_{q<\Lambda}\frac{d^dq}{(2\pi)^d}\times\vartheta\left(2\mathbf{q}\right)\cdot\left(\frac{1}{2}\mathbf{k}+\mathbf{q}\right)\cdot\left(\frac{1}{2}\mathbf{k}-\mathbf{q}\right)\times h\left(\frac{1}{2}\mathbf{k}+\mathbf{q},\frac{1}{2}\omega+\Omega\right)h\left(\frac{1}{2}\mathbf{k}-\mathbf{q},\frac{1}{2}\omega-\Omega\right), \tag{5}$$

where $G_0(\mathbf{k},\omega)=1/(\nu k^2+\kappa k^4-i\omega)$ is the bare propagator or the Green function for the diffusion equation, and Λ (can be set to 1) is related to the microscopic cutoff. Equation (5) is a convenient starting point for a perturbative calculation of $h(\mathbf{k},\omega)$ in powers of λ. A diagrammatic representation can be set up for Eq. (5) as indicated in Fig. 1. The diagrammatic representation is quite standard with $\rightarrow$ indicating the bare

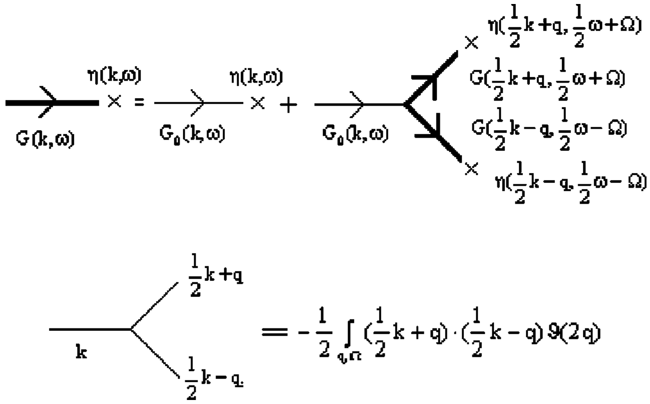

FIG. 1. Diagrammatic representation of Eq. (5).

propagator $G_0(k,\omega)$ and $\times$ depicting the noise $\eta(\mathbf{k},\omega)$. In the usual iterative perturbative scheme, $h(\mathbf{k},\omega)$ in the right-hand side of Eq. (5) is replaced by Eq. (5) itself up to $O(\vartheta^2)$. The renormalization of the parameters in Eq. (4) can be obtained from appropriate vertex functions. As in Refs. [6,7,18], the effective propagator $G(\mathbf{k},\omega)\equiv h(\mathbf{k},\omega)/\eta(\mathbf{k},\omega)$ gives the renormalization of the tension ν and the diffusion κ. The effective noise, obtained from $\langle h^*(\mathbf{k},\omega)\cdot h(\mathbf{k},\omega)\rangle = 2DG(\mathbf{k},\omega)G(-\mathbf{k},-\omega)$, gives the renormalization of the disorder D. The procedure adopted here to calculate the complete RG flow for Eq. (4) is similar to Refs. [6] and [7]. However, the calculations here are much more tedious. These result from the fact that we must keep terms up to order k^4 in the perturbative series and contain the long-range part of $\vartheta(\mathbf{k})$ in the calculation.

Due to the Galilean invariance of Eq. (4) , λ_0 is not renormalized. Since the RG transformation is analytic in nature, λ_ρ is also not renormalized; so we got the following scaling exponents identity as in Refs. [11] and [13]:

$$\chi+z=2-\rho, \tag{6}$$

where $\rho=0$ for λ_0 flow. Equation (6) is the result of a one-loop approximation [12].

The flow equations, we obtained, for ν, κ, D, λ_0, and λ_ρ are the following:

$$\frac{d\nu}{dl}=\nu\left[z-2+K_d\frac{\vartheta(2)\vartheta(1)}{\nu}\times\frac{\nu\{2-d-3f(1)\}+\kappa\{4-d-3f(1)\}}{4d(\nu+\kappa)^3}\right], \tag{7}$$

$$\frac{d\kappa}{dl}=\kappa\left[z-4+K_d\frac{\vartheta(2)\vartheta(1)}{\kappa}\times\frac{a_0\nu^3+a_1\nu^2\kappa+a_2\nu\kappa^2+a_3\kappa^3}{16d(d+2)(\nu+\kappa)^5}\right], \tag{8}$$

$$\frac{dD}{dl}=D\left[z-2\chi-d+K_d\frac{D\vartheta(2)^2}{4(\nu+\kappa)^3}\right], \tag{9}$$

$$\frac{d\lambda_0}{dl}=\lambda_0(z+\chi-2), \tag{10}$$

$$\frac{d\lambda_\rho}{dl}=\lambda_\rho(z+\chi-2+\rho), \tag{11}$$

where $K_d=S_d/(2\pi)^d=[S_{d-1}/(2\pi)^d]\int_0^\pi d\theta\sin^{d-2}\theta$, S_d is the surface area of the d-dimensional unit sphere, and $f(a) =\partial\ln\vartheta(k)/\partial\ln k|_{k=a}$. The polynomials a_i $(i=0,1,2,3)$ are given by

$$a_0=3d(d-2)+3f(1)(5d-2),$$

$$a_1=11d^2-24d-20+3f(1)(19d+2),$$

$$a_2=13d^2-40d-60+3f(1)(23d-2),$$

$$a_3=5d^2-22d-16+3f(1)(9d-6). \tag{12}$$

The flow equations (7)–(11) contain all information regarding the dynamic scaling behaviors of Eq. (4). We can solve them for the DRG fixed points of Eq. (4) (point at which parameters are unchanged under rescaling) and determine the values of the corresponding critical exponents.

It can be seen that the flow equations (7)–(9) can shrink to the flow equations of the original noisy KS equation discussed in Ref. [18] for the local case of $\lambda_\rho=0$ with $f(1)=0$. At the same time, taking $\kappa=0$, the flow equations for the nonlocal KPZ equation of Mukherji and Bhattacharjee [11] can be arrived at. So we can expect that all standard results of the noisy KS equation and the nonlocal KPZ equation can be obtained from Eqs. (7)–(9) and (10) or (11), respectively. In fact, our following calculations and discussion confirm these predictions.

When studying the above RG flow, it is convenient to define the coupling constants $U_{0,\nu}^2\equiv(K_dD\lambda_0^2)/\nu^3$, $U_{\rho,\nu}^2\equiv(K_dD\lambda_\rho^2)/\nu^3$, $R_\nu=U_{0,\nu}/U_{\rho,\nu}$, and $f_\nu=\kappa/\nu$. From Eqs. (7)–(11), we can obtain the flow equations for $U_{0,\nu}$, $U_{\rho,\nu}$, f_ν, R_ν,

$$\frac{dU_{0,\nu}}{dl}=U_{0,\nu}\left[\frac{2-d}{2}+\frac{b_0U_{0,\nu}^2+b_1U_{0,\nu}U_{\rho,\nu}+b_2U_{\rho,\nu}^2}{8d(1+f_\nu)^3}\right], \tag{13}$$

$$\frac{dU_{\rho,\nu}}{dl}=U_{\rho,\nu}\left[\frac{2-d+2\rho}{2}+\frac{b_0U_{0,\nu}^2+b_1U_{0,\nu}U_{\rho,\nu}+b_2U_{\rho,\nu}^2}{8d(1+f_\nu)^3}\right], \tag{14}$$

$$\frac{dR_\nu}{dl}=-\rho R_\nu, \tag{15}$$

$$\frac{df_\nu}{dl}=-2f_\nu+\frac{1}{16d(d+2)(1+f_\nu)^5}[(c_0+c_1f_\nu+c_2f_\nu^2+c_3f_\nu^3+c_4f_\nu^4)U_{0,\nu}^2+(e_0+e_1f_\nu+e_2f_\nu^2+e_3f_\nu^3+e_4f_\nu^4)U_{0,\nu}U_{\rho,\nu}+(g_0+g_1f_\nu+g_2f_\nu^2+g_3f_\nu^3+g_4f_\nu^4)2^{-\rho}\cdot U_{\rho,\nu}^2], \tag{16}$$

where polynomials b_i, c_i, e_i, and g_i $(i=0,1,2,\ldots)$ are given, respectively, by

$$b_0=4d-6+3f_\nu(d-4),$$

$$b_1=(5d-6)2^{-\rho}-3(2-d+3\rho)-3f_\nu[(4-d)2^{-\rho}+(4-d+3\rho)],$$

$$b_2=[(3+2^{-\rho})d-6-9\rho]2^{-\rho}-3f_\nu(4-d+3\rho)2^{-\rho}; \tag{17}$$

$$c_0=3d(d-2),\quad c_1=15d^2-24d-36,$$

$$c_2=25d^2-48d-124,$$

$$c_3=17d^2-38d-96,\quad c_4=4d^2-8d-32; \tag{18}$$

$$e_0=c_0(1+2^{-\rho})-3\rho(21d+18),$$

$$e_1=c_1(1+2^{-\rho})-3\rho(23d+10),$$

$$e_2=c_2(1+2^{-\rho})-3\rho(35d+22),$$

$$e_3=c_3(1+2^{-\rho})-3\rho(21d+18),$$

$$e_4=c_4(1+2^{-\rho})-12\rho(d+2); \tag{19}$$

and

$$g_0=c_0-3\rho(5d-2),\quad g_1=c_1-3\rho(23d+10),$$

$$g_2=c_2-3\rho(35d+22),$$

$$g_3=c_3-3\rho(21d+18),\quad g_4=c_4-12\rho(d+2). \tag{20}$$

In the nonlocal KPZ situation [11], the coupling constants one needs to study, are $U_{0,\nu}$, and $U_{\rho,\nu}$. In our case, however, the flow for $U_{0,\nu}$ and $U_{\rho,\nu}$ is affected by the additional coupling f_ν, which probes the relevance at large distances of surface diffusion with respect to surface tension.

The variables $U_{0,\nu}$, $U_{\rho,\nu}$, and f_ν are convenient to analyze in the cases where κ is smaller than ν. On the other hand, when ν is flowing towards zero the natural coupling constants are $U_{0,\kappa}^2\equiv(K_dD\lambda_0^2)/\kappa^3=U_{0,\nu}^2/f_\nu^3$, $U_{\rho,\kappa}^2\equiv(K_dD\lambda_\rho^2)/\kappa^3=U_{\rho,\nu}^2/f_\nu^3$, $R_\kappa=U_{0,\kappa}/U_{\rho,\kappa}$, and $f_\kappa=\nu/\kappa=f_\nu^{-1}$ [18], for which the RG flow becomes

$$\frac{dU_{0,\kappa}}{dl}=\frac{U_{0,\kappa}}{2}\Bigg\{(8-d)-\frac{1}{16d(d+2)(1+f_\kappa)^5}[(b_0'+b_1'f_\kappa+b_2'f_\kappa^2+b_3'f_\kappa^3)U_{0,\kappa}^2+(c_0'+c_1'f_\kappa+c_2'f_\kappa^2+c_3'f_\kappa^3)U_{0,\kappa}U_{\rho,\kappa}+(e_0'+e_1'f_\kappa+e_2'f_\kappa^2+e_3'f_\kappa^3)2^{-\rho}U_{\rho,\kappa}^2]\Bigg\}, \tag{21}$$

$$\frac{dU_{\rho,\kappa}}{dl}=\frac{U_{\rho,\kappa}}{2}\Bigg\{(8-d+2\rho)-\frac{1}{16d(d+2)(1+f_\kappa)^5}[(b_0'+b_1'f_\kappa+b_2'f_\kappa^2+b_3'f_\kappa^3)U_{0,\kappa}^2+(c_0'+c_1'f_\kappa+c_2'f_\kappa^2+c_3'f_\kappa^3)U_{0,\kappa}U_{\rho,\kappa}+(e_0'+e_1'f_\kappa+e_2'f_\kappa^2+e_3'f_\kappa^3)2^{-\rho}U_{\rho,\kappa}^2]\Bigg\}, \tag{22}$$

$$\frac{dR\kappa}{dl}=-\rho R_\kappa, \tag{23}$$

$$\frac{df_\kappa}{dl}=2f_\kappa-\frac{1}{16d(d+2)(1+f_\kappa)^5}[(c_0f_\kappa^4+c_1f_\kappa^3+c_2f_\kappa^2+c_3f_\kappa+c_4)U_{0,\kappa}^2+(e_0f_\kappa^4+e_1f_\kappa^3+e_2f_\kappa^2+e_3f_\kappa+e_4)U_{0,\kappa}U_{\rho,\kappa}+(g_0f_\kappa^4+g_1f_\kappa^3+g_2f_\kappa^2+g_3f_\kappa+g_4)2^{-\rho}\cdot U_{\rho,\kappa}^2]. \tag{24}$$

with the polynomials b_i', c_i', and e_i' $(i=0,1,2,3)$,

$$b_0'=11d^2-74d-48,\quad b_1'=31d^2-136d-180,$$

$$b_2'=29d^2-80d-60,\quad b_3'=9d^2-18d; \tag{25}$$

$$c_0'=(7d^2-82d-48)(1+2^{-\rho})+8d(d+2)-9\rho(9d-6),$$

$$c_1'=(23d^2-154d-180)(1+2^{-\rho})+16d(d+2)-9\rho(23d-2),$$

$$c_2'=(25d^2-88d-60)(1+2^{-\rho})+8d(d+2)-9\rho(19d+2),$$

$$c_3'=(9d^2-18)(1+2^{-\rho})-9\rho(21d+18); \tag{26}$$

and

$$e_0'=15d^2-66d-48-4d(d+2)2^{-\rho}-9\rho(9d-6),$$

$$e_1'=39d^2-120d-180-8d(d+2)2^{-\rho}-9\rho(23d-2),$$

$$e_2'=33d^2-72d-60-4d(d+2)2^{-\rho}-9\rho(19d+2),$$

$$e_3'=(9d^2-18)-9\rho(5d-2). \tag{27}$$

The expressions of the b_i, c_i, e_i, and g_i $(i=0,1,2,\ldots)$ in Eq. (24) are the same as those in Eq. (16). The flow equations (13)–(16) and (21)–(24) can, respectively, reduce to the corresponding flow equations of the noisy KS equation in the case of $\lambda_\rho=0$ [18]. Note also that Eqs. (13)–(16) and Eqs. (21)–(24) describe the same system and, accordingly, we can analyze the effect of the RG transformation using any set of them.

IV. CALCULATION AND DISCUSSION

Equations (15) and (23) rule out the existence of any off-axis fixed point in the $(U_{0,\nu},U_{\rho,\nu})$ space or in the $(U_{0,\kappa},U_{\rho,\kappa})$ (except for the trivial case $\rho=0$). From Eqs.

(13)–(16), we find that there are only two sets of axial points in the two dimensional $(U_{0,\nu},U_{\rho,\nu})$ space: for the short range,

$$U_{0,\nu}^{*2}=\frac{4d(d-2)(1+f_\nu)^3}{4d-6+3(d-4)f_\nu},$$

$$U_{\rho,\nu}^{*2}=0, \tag{28}$$

with

$$\frac{df_\nu}{dl}=-2f_\nu+\frac{1}{16d(d+2)(1+f_\nu)^5}\times(c_0+c_1f_\nu+c_2f_\nu^2+c_3f_\nu^3+c_4f_\nu^4)U_{0,\nu}^2, \tag{29}$$

$$\chi+z=2; \tag{30}$$

and for the long range,

$$U_{0,\nu}^{*2}=0,$$

$$U_{\rho,\nu}^{*2}=\frac{4d(d-2-2\rho)(1+f_\nu)^3}{[(3+2^{-\rho})d-6-9\rho]2^{-\rho}+3f_\nu(d-4-3\rho)2^{-\rho}}, \tag{31}$$

with

$$\frac{df_\nu}{dl}=-2f_\nu+\frac{1}{16d(d+2)(1+f_\nu)^5}\times(g_0+g_1f_\nu+g_2f_\nu^2+g_3f_\nu^3+g_4f_\nu^4)2^{-\rho}U_{\rho,\nu}^2, \tag{32}$$

$$\chi+z=2-\rho. \tag{33}$$

The first set for the short-range case of $\lambda_\rho=0$ SR corresponds to the fixed points of the local noisy KS equation, whose RG flow and scaling properties have been completely and adequately discussed by Cuerno and Lauritsen in Ref. [18]. In fact, in the case of $f\nu=0$ (or $\kappa=0$), Eqs. (28) and (31), respectively, give the two fixed points obtained in Ref. [11] for the nonlocal KPZ equation. In the present work, we focus our attention to the second set for the long range case LR to study the effects of long-range interactions on the noisy KS equation.

We will next discuss the fixed points of the long-range part expressed in Eqs. (31)–(33), and determine the values of the critical exponents χ and z.

Setting $d\nu/dl=df_\nu/dl=0$ and taking $\lambda_0=0$, Eqs. (7) and (32), respectively, become

$$z=2+U_{\rho,\nu}^{*2}\frac{[(d-2-3\rho)+f_\nu(d-4-3\rho)]2^{-\rho}}{4d(1+f_\nu)^3} \tag{34}$$

and

$$2f_\nu-\frac{1}{16d(d+2)(1+f_\nu)^5}(g_0+g_1f_\nu+g_2f_\nu^2+g_3f_\nu^3+g_4f_\nu^4)2^{-\rho}U_{\rho,\nu}^{*2}=0. \tag{35}$$

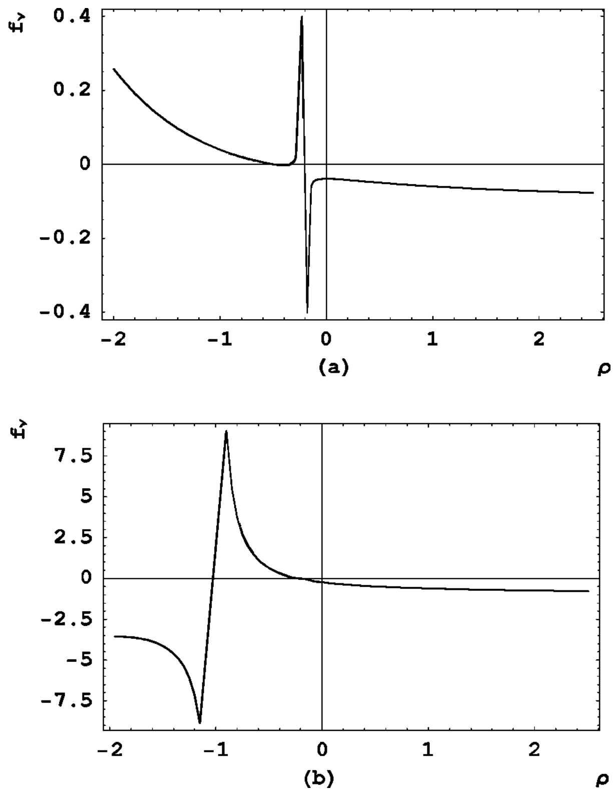

FIG. 2. In $d=1$, the two real number solutions of Eq. (39) as a function of the parameter ρ are shown in (a) and (b), respectively.

Substituting $U_{\rho,\nu}^{*2}$ of Eq. (31) into Eqs. (34) and (35) yields the following equations:

$$z=2+\frac{(d-2-2\rho)[(d-2-3\rho)+f_\nu(d-4-3\rho)]}{(3+2^{-\rho})d-6-9\rho+3f_\nu(d-4-3\rho)}, \tag{36}$$

and

$$(d-2-2\rho)(g_0+g_1f_\nu+g_2f_\nu^2+g_3f_\nu^3+g_4f_\nu^4)-8(d+2)f_\nu(1+f_\nu)^2[(3+2^{-\rho})d-6-9\rho+3f_\nu(d-4-3\rho)]=0. \tag{37}$$

The complexities of Eqs. (36) and (37) prevent us from getting simply analytical expressions for f_ν and z. For definite d and ρ, however, we can determine z by solving Eq. (37) numerically and then substituting the value of. f_ν obtained into Eq. (36). The corresponding value of χ can be obtained from Eq. (33).

In the cases of $d=1$ and 2, we solve Eq. (37) numerically and find that for a definite ρ, Eq. (37) has two real number solutions. The values of f_ν calculated in the range of $-2<\rho<2.5$ are shown in Fig. 2 (for $d=1$) and Fig. 3 (for $d=2$), respectively. It can be seen that in both $d=1$ and 2, one

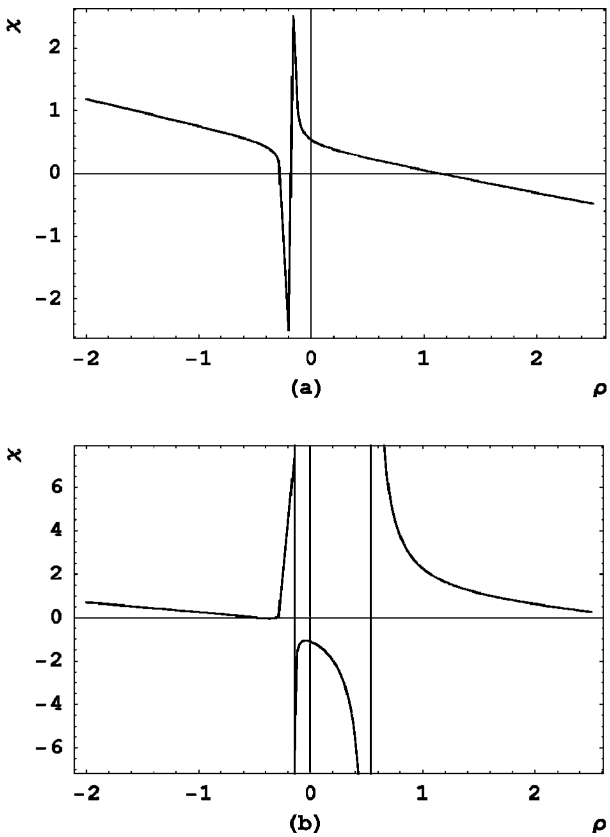

FIG. 3. In $d=1$, the calculated values of χ as a function of the parameter ρ. (a) and (b) correspond to Figs. 2(a) and 2(b), respectively.

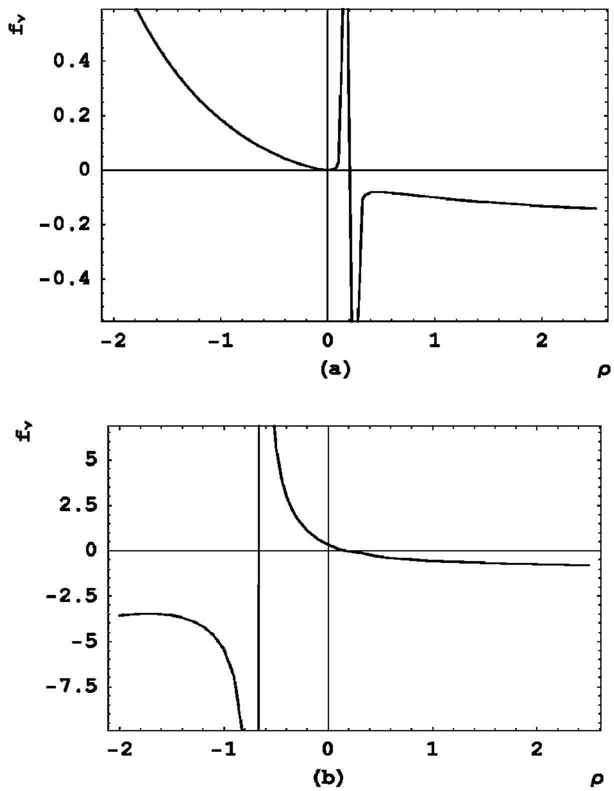

FIG. 4. In $d=2$, the two real number solutions of Eq. (39) as a function of the parameter ρ are shown in (a) and (b), respectively.

of the two solutions is very close to $f_\nu=0$, cf. Figs. 2(a) and 3(a), which, we think, corresponds to the fixed point of the nonlocal KPZ equation [11]. Another solution is related to the new fixed point that does not exist in the original noisy KS equation. The values of χ corresponding to f_ν indicated in Figs. 2 and 3 are shown in Figs. 4 and 5, respectively. The divergent points turn out to be an artifact of the one-loop approximation [11].

From Eqs. (13) and (14), we can see that when $U_{\rho,\nu}=0$, the SR fixed point is stable for $d<2$; on the other hand when $U_{0,\nu}=0$, the LR fixed point for $d<2+2\rho$ is stable. It indicates that for the long-range part, the stable range is $\rho>-1/2$ for $d=1$, and $\rho>0$ for $d=2$. According to the discussions of Refs. [11] and [12], for the positive values of ρ, the effective nonlinearity $U_{\rho,\nu}$ is dominant over $U_{0,\nu}$; thus the phase in all space ($U_{\rho,\nu}$, $U_{0,\nu}$) except for $U_{\rho,\nu}=0$ is determined by the long-range λ_ρ term in Eq. (4), while for negative values of ρ, the LR fixed point is irrelevant on the ground that $U_{0,\nu}$ is dominant over $U_{\rho,\nu}$. So the short-range term in Eq. (4) that describes the nonlinearity of the original noisy KS equation determines the surface behavior in all space ($U_{\rho,\nu}$, $U_{0,\nu}$). As in Refs. [11] and [12], in Figs. 4 and 5, we call the range of $\chi>0$ the long-range rough phase, and the range of $\chi<0$ the long-range smooth phase.

Mukherji and Bhattacharjee [11] gave the expression of χ related to the long range fixed points of the nonlocal KPZ equation by

$$\chi=-\rho-\frac{(d-2-2\rho)(d-2-3\rho)}{d(3+2^{-\rho})-6-9\rho}. \tag{38}$$

In the $d=1$ and 2, the graphic expressions of Eq. (38) are given in Fig. 6. Obviously, Figs. 6(a) and 6(b) are very similar to Figs. 4(a) and 5(a), respectively. In fact, we can get Eq. (38) from Eqs. (33) and (36) by setting $f_\nu=0$. These results imply that both our analytical and numerical calculations can well match the results of Mukherji and Bhattacharjee in Ref. [11].

In addition, in Figs. 4(b) and 5(b), the values of χ at the points $\rho=0$ are -1.122 (for $d=1$) and -0.493 (for $d=2$), respectively. These values of χ are exactly equal to the corresponding values that Cuerno and Lauritsen got in Ref. [18]. Physically, these results are reasonable since in the case of $\rho=0$, Eq. (4) actually does not contain the long-range interaction in its nonlinear term. As a result, it presents the scaling properties of the original noisy KS equation.

If observing in the ($U_{0,\kappa}$, $U_{\rho,\kappa}$, f_κ) space, we can get, from Eqs. (21)–(23), the fixed point for the long-range part

$$U_{0,\kappa}^{*2}=0,$$

$$U_{\rho,\kappa}^{*2}=\frac{16d(d+2)(8-d+2\rho)(1+f_\kappa)^5}{(e_0'+e_1'f_\kappa+e_2'f_\kappa^2+e_3'f_\kappa^3)2^{-\rho}}, \tag{39}$$

with

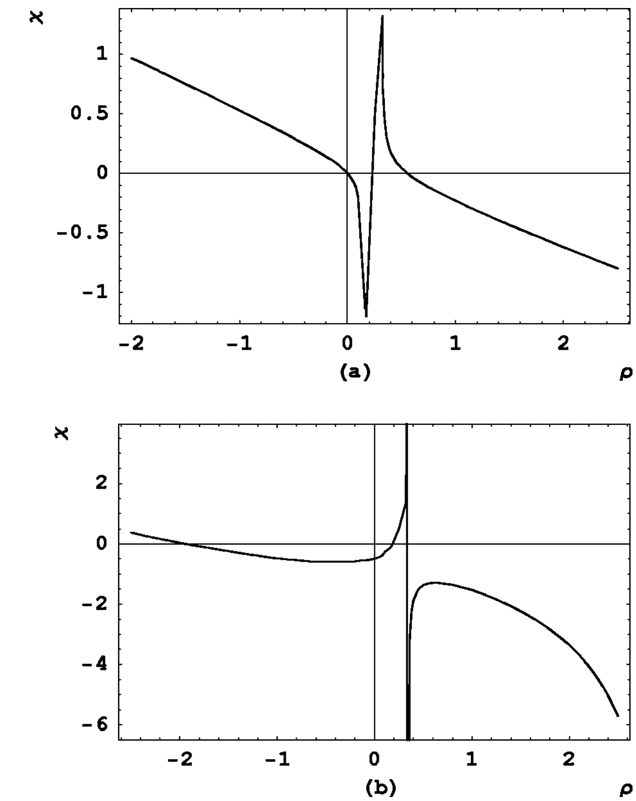

FIG. 5. In $d=2$, the calculated values of χ as a function of the parameter ρ. (a) and (b) correspond to Figs. 4(a) and 4(b), respectively.

$$\frac{df_\kappa}{dl}=2f_\kappa-\frac{2^{-\rho}(g_0f_\kappa^4+g_1f_\kappa^3+g_2f_\kappa^2+g_3f_\kappa+g_4)}{16d(d+2)(1+f_\kappa)^5}U_{\rho,\kappa}^{*2}. \tag{40}$$

The dynamic exponents z can be expressed with f_κ and $U_{\rho,\kappa}^{*2}$

$$z=2+U_{\rho,\kappa}^{*2}\frac{[(d-2-3\rho)f_\kappa+(d-4-3\rho)]2^{-\rho}}{4df_\kappa(1+f_\kappa)^3}. \tag{41}$$

Substituting $U_{\rho,\kappa}^{*2}$ of Eq. (39) into Eqs. (40) and (41), and then setting $df_\kappa/dl=0$, we have the following equations for f_κ and z;

$$2f_\kappa(e_0'+e_1'f_\kappa+e_2'f_\kappa^2+e_3'f_\kappa^3)-(8-d+2\rho)(g_0f_\kappa^4+g_1f_\kappa^3+g_2f_\kappa^2+g_3f_\kappa+g_4)=0, \tag{42}$$

$$z=2+\frac{8(d+2)(1+f_\kappa)^2[(d-2-3\rho)f_\kappa+(d-4-3\rho)]}{(g_0f_\kappa^4+g_1f_\kappa^3+g_2f_\kappa^2+g_3f_\kappa+g_4)}. \tag{43}$$

In the same way, for definite d and ρ we can calculate the values of z and χ by solving Eq. (42) numerically and then substituting the values of f_κ obtained into Eq. (43). Our calculations show that the same results can be obtained in the $(U_{0,\kappa},U_{\rho,\kappa},\ f_\kappa)$ variables. In fact, the polynomial g_4 in Eq. (42) is given by $g_4=4[d^2-2d-8-3\rho(d+2)]$, from which

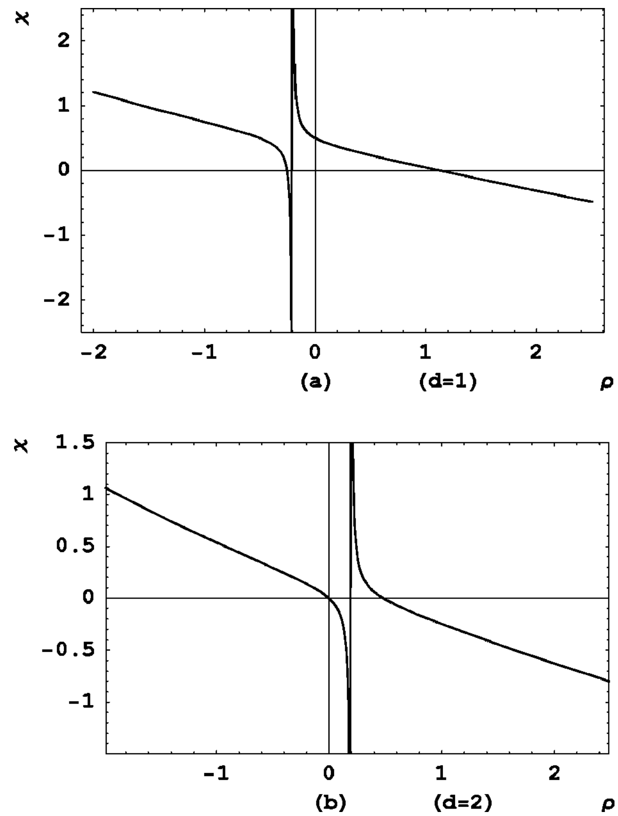

FIG. 6. The values of χ as a function of the parameter ρ given by Eq. (39). (a) and (b) are for $d=1$ and $d=2$, respectively.

we can easily arrive at the results that the solution of Eq. (42) is $f_\kappa=0$ in the case of $\rho=-1.0$, for $d=1$ and $\rho=-2/3$, for $d=2$. These results respectively correspond to the divergent values of f_ν in Figs. 2(a) and 3(a). As mentioned previously, we can analyze the system equivalently in the $(U_{0,\nu},\ U_{\rho,\nu},\ f_\nu)$ or $(U_{0,\kappa},\ U_{\rho,\kappa},\ f_\kappa)$ space.

V. CONCLUSIONS

In the present paper, we introduced the long-range nonlinearity of Mukherji and Bhattacharjee into the noisy KS equation. By DRG analysis and numerical calculations, we show that the existence of the long-ranged interactions in the noisy KS equation, as in the KPZ equation, can produce new stable fixed points with varying scaling exponents that depend on the long-range interaction parameter ρ and the substrate dimension d. The fixed points for the short-range part correspond to the original noisy KS equation. The fixed points for the long-range part are the new fixed points that are generated by the nonlocal nonlinearity in the noisy KS equation. For the positive values of ρ, the effective nonlineaity $U_{\rho,\nu}$ determines the behavior of growing surface; for the negative values of ρ, however, the long-range nonlinearity is irrelevant as a result of fact that $U_{0,\nu}$ is dominant over $U_{\rho,\nu}$. The short-range term λ_0 determines the growth process. Depending on the values of ρ and d, different phases can be obtained.

By setting the parameter $\kappa=0$, the nonlocal noisy KS equation smoothly shrinks to the nonlocal KPZ equation originally proposed and discussed by Mukherji and Bhattacharjee [11]. On the other hand, from the nonlocal noisy KS equation, we can get the standard noisy KS equation by setting $\lambda_{\rho}=0$ in the kernel function. Our discussions and calculations can give the results that are obtained from these two equations. So our present work also has general interest. Since it is not clear as to which experimental system really possesses the long-range interaction expressed in Eq. (4), examining experimentally our results obtained here will be interesting.

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